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Cutting Entanglement

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The concept of entanglement is sometimes introduced in the published literature where it is unnecessary. The cases of Newtonian-, special relativistic-, and quantum mechanical conservation of linear and angular momentum are here treated without entanglement, as is the two-slit interference experiment. A treatment of a tropic where entanglement *is* necessary, q-bits, is then given. This article was prepared in the interest of helping students studying quantum mechanics.

1. Introduction

This article is written in the spirit of not unnecessarily complicating things when teaching students, especially when quantum mechanics is concerned. The systems discussed are idealised, in the sense that they are isolated.

Let us start with the laws of conservation of energy, momentum, and angular momentum. In Newtonian-, and Einsteinian special relativistic systems, these laws hold. So consider, in a Newtonian system, a device we shall call a "bomb", with mass M, which is isolated and initially at rest in an inertial reference frame. We can detonate it so that it splits into only two pieces with masses m' and m'', the sum of which is equal to M. Then we know that the momentum conservation law states that the momentum of m' is equal and opposite to that of m''. So m' and m'' are related together as a system, so long as they are not interfered with, from one end of the universe to another, in distance and in time. And if the momentum of m' is measured, then the momentum of m'' must always be equal and opposite. So if an observer 1 in the inertial frame is the first to make a measurement, he correctly predicts the result of observer 2, no matter how far away they are separated in time or distance. This is also true for special relativity, though the formalism is a little different.

Do we call this *entanglement*? No, we do not even question it, because it is a consequence of the conservation law. Do we talk about *preprogramming,* as in some discussions of entanglement? Only if we are talking about the bomb designers, that is, the arranged properties of the two parts of the bomb.

If m' has an angular momentum imposed on it, we know that m'' has an equal and opposite angular momentum, because of the conservation law. The same comments and question, and the answer above, apply. Especially, special relativity applies.

2. Relativity and quantum mechanics

We are now going to consider relativistic problems, to which quantum mechanics applies. It should be pointed out that quantum mechanics and special relativity are compatible. Even stimulated emission is compatible with special relativity, though Einstein used what might be called a "semiclassical" argument in his paper introducing it.

2.1 Pair annihilation

When an electron-positron pair annihilates, positronium is formed first, and the particle spins are opposed, so the initial angular momentum is zero. Therefore the sum of the spins of the resulting photons must also be zero, leading to them having opposite spins, each of magnitude 1. So if we detect a spin of +1, the other photon must have spin -1. How is this result different from the two spinning bomb parts? It is not: we have a similar expanding system brought about by the conservation law.

2.2 Pair production

The photon producing the electron-positron pair has a spin of ±1. Therefore the (spin ½) electron and the positron must have the same spin direction, to give a total spin of ±1. Again, this is, by extension, no different from our bomb parts result. Yet these results are often unnecessarily referred to as entanglement. Entanglement is much more than this!

We examine now the quantum mechanical treatment of a created "electron" pair, travelling in opposite directions, subject to conservation of angular momentum so that the spins are either both "up" or both "down". Our detector can detect either configuration, and distinguish between them. The beams have been filtered so that only spin up is detected. In what follows, operators will be denoted by Roman letters (thus: a, A) and ordinary quantities and scalars are denoted by italics (thus: *a*, *A*).

Let one of the "electron beams" be travelling along the *x* direction, with spins up in the *z* direction. The spin states are represented by the column vectors $|\,1\;0\,\rangle$ (spin up) and $|\,0\;1\,\rangle$ (spin down). In the situation described, our detectors will detect only $|\,1\;0\,\rangle$. Now a detector is rotated by an angle α. This is equivalent to a unitary transformation of the S_z spin operator, so the eigenvalues are the same. However, since the new state detected is a mixture of the S_z and S_y states, our detector can and will now detect spin down as well as spin up. The same must be true of the detector in the other beam, if it is similarly rotated. The states $|\,1\;0\,\rangle$ and $|0\;1\,\rangle$ will have different probabilities of being detected, *but these probabilities will be the same for each beam*. This is considered by some to be the quantum mechanical contribution to "entanglement", but it is only state mixing. Yes, it is strange, but so were potential barrier penetration and the zero orbital angular momentum of all *s*-state atomic orbitals. The quantum mechanical result for the beams obeys the appropriate law of angular momentum conservation and the rules of quantum mechanics.

2.3 The two-slit experiment

This experiment has been performed with photons, electons and atoms. Prof. Roy Glauber, discoverer of the coherent state representation and developer of a new theorem in quantum mechanics, made the very relevant comment that "The things that interfere in quantum mechanics . . . are probability amplitudes for certain events" [1]. The following discussion is on photons but most of it is applicable to other particles, *mutatis mutandis*.

Let the slits have a separation *d*. Let the wavelength of the incident radiation be λ. According to the uncertainty principle, $\Delta p.\Delta q \approx h$, where *p* is the momentum, *q* is the appropriate displacement and *h* is Planck's constant. Now $\Delta q = d$, so that $\Delta p \approx h/d$ in the direction perpendicular to the incident momentum $p = h/\lambda$. Therefore the sine of the small angle between the resultant momentum and the incident momentum is approximately $\Delta p/p = \lambda/d$. This is the separation angle between two bright fringes. Is it necessary to invoke entanglement here? Obviously not.

Better is to come. An experiment after the style of that by Basano and Ottonello [2] has been performed with two identical frequency stabilised lasers, each focussed on only one (different) slit of the pair. One laser was run below threshold, thus emitting amplified 'thermal' noise which has a Bose-Einstein distribution over the number states of the representative harmonic oscillator. The other laser was operated well above threshold, giving the "coherent state" distribution for the harmonic oscillator. Of course fringes occurred! A photon counting experiment on the central maximum of the fringe pattern should show the distribution for "signal plus noise" which is well known.

In all of the above discussion, there was no need to draw on the concept of entanglement, yet one can find discussions of these situations elsewhere in the literature where it *is* mentioned unnecessarily. Why, when it is clearly not required in these situations?

3. Entanglement, decoherence and quantum computation

Thus far we have largely focussed our attention on what entanglement *is not*. This context allows us to sharpen our degree of understanding of what entanglement *is*, in a purely quantum setting divorced from the conservation-induced correlations discussed earlier.

To this end, let us return our consideration to the electron-electron pair. Suppose that a relevant conservation law implies the electron-electron pair to have opposite spin projections with respect to the axis of quantization imposed by a given uniform external magnetic field. Quantum mechanically, and ignoring an irrelevant normalization constant, assume the two-electron system to be specified by the pure-state, two-body, state vector $|1,-1\rangle + |-1, 1\rangle$. Let us whimsically picture this as a coherent superposition of (i) a physicist pointing their left thumb up and their right thumb down, with (ii) the same physicist pointing their left thumb down and their right thumb up (Fig. 1).

Fig. 1. Whimsical representation of an "entangled" state for a two electron spin system.

Such a state is said to be entangled because we have lost the classical notion that meaning can be given to "the state" of any component of the whole system. That is, replacing electron spins with directions of thumbs, we can no longer speak of "the state" of the left thumb, or "the state" of the right thumb – thus the system is entangled, in precisely the sense originally envisaged by Schrödinger.

The perturbing effects of the environment upon such a superposition can have the effect of imposing classicality, in the sense that coupling to the environment can be viewed as a form of measurement, which destroys entanglement by detecting the state of either thumb/electron.

Yet how much is lost in the process! An arbitrary entangled state of the electron-positron pair can be described by three real numbers (the complex coefficients of $|1,-1\rangle$ and $|-1,1\rangle$ yield four real numbers, with the demand for normalization reducing this to three real numbers). The corresponding classical state is described by either a zero or a 1, depending on whether one has the classical state $|1,-1>$ or the classical state $|-1,1>$.

This huge discrepancy between the amount of information needed to describe the classical versus the quantum state, and which becomes exponentially more vast as the number of quantum particles increases, can be loosely encapsulated in the phrase "Hilbert space is big." As has been emphasized by Bennet and Vincenzo (2000) [3], this fact lies at the heart of both the power of quantum computation (here, we identify entangled two-body or n-body quantum systems with quantum bits, i.e. "qubits") and the challenge in building such devices.

The real world intervenes to make our job of building a quantum computer more challenging. The effects of entanglement do not last very long in all but the most isolated systems, with the environment serving to "decohere" delicate quantum-mechanical entangled superpositions into merely classical states. The means of protecting quantum systems from such perturbing effects of the environment, which destroy quantum entanglement and cripple quantum computation, remain the subject of active research and one of the most exciting contemporary consequences of the phenomenon of quantum-mechanical entanglement. In this context, we re-iterate a central point of these discussions, that one's understanding of the essence of entanglement may be sharpened by drawing a clear distinction between conservation-induced correlations (which may occur in both classical and quantum systems) and purely quantum phenomena which have no classical counterpart.

4. Conclusion

In all but the last of the cases we have discussed, it is unnecessary to introduce the concept of entanglement: conservation laws and the principles of quantum mechanics suffice. Students have difficulties when being introduced to quantum mechanics, and also in continuing to study it. Simplicity and reassurance help in dealing with these difficulties. Of course, things change when dealing with the "Schrodinger's Cat" states used in "q-bit" studies, where entanglement really *is* necessary.

References

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[2] Basano L and Ottonello P 2000 Am. J. Phys. **68** 245
[3] Bennett C H and Vincenzo D P 2000 Nature **404** 247-255