# Performance evaluation for a SQUID based Metal detection system

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A threshold detector and an energy detector were developed for a SQUID based metal detection system. With the use of statistical analysis, the detection performance of the two methods were modelled and compared. Suitable detection thresholds were specified, according to our desired performance requirements. The analytic results were compared to the performance of the detectors operating on sampled data from a current laboratory prototype system.

## 1. Introduction

Two high-temperature superconducting quantum interference device (SQUID) vector magnetometers, with a noise performance of 300 *f*T / √Hz, are used to detect small soft-steel contaminants in food products passing on a conveyor underneath the sensors. The magnetometers are separated horizontally by a baseline, *d*[1], and oriented in line with the movement of the products. As contaminated samples pass the sensors, they produce a characteristic signal in each, with the second sensor's output delayed but identical to the first. Any fluctuations in the external magnetic field are highly uniform across the base-line of the detector and constitute a common-mode interference source. Thus the subtraction of one sensor's output from the other, known as gradiometry, can be used to remove most of this external interference. The resulting signal *g[n]* is band pass filtered to remove any arbitrary DC offset and remaining out-of-band interference, and serves as input signal for the detector. Two detectors have been developed. The 'Single Threshold Detector' (STD), which decides that a contamination is present if any one sample exceeds a threshold, or $T(x) = |g[n]| > \gamma_1$,

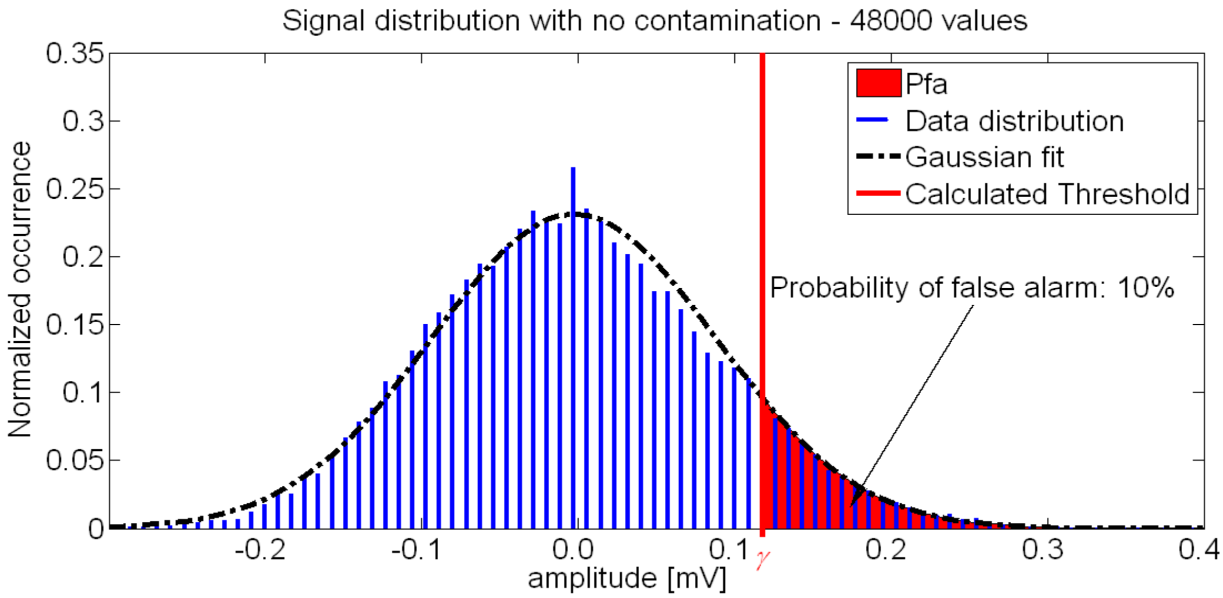


Figure 1. Data distribution over 240 sec without contamination; threshold calculated for $P_{FA}$ = 10%.

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[1] d = 5 cm for the laboratory prototype system used for all data acquisition during this project

and the ‘Energy Detector’ (ED), which detects a contaminant if the energy in the signal over a set of samples exceeds a threshold, or $T(x) = \sum_{n=1}^{N} g[n]^2 > \gamma_2$.

In order to set the thresholds $\gamma_1$ and $\gamma_2$ to an optimal level it is important to know the distribution of the signal *g[n]*. Data from our prototype system suggests that in the uncontaminated state, $H_0$, the signal is Gaussian (Figure 1). We can write *g[n]=w[n]*, where *w[n]* is a random variable with a Gaussian distribution, variance $\sigma^2$ and mean value $\mu=0$. In state $H_1$, the presence of a contaminated sample adds the signal *s[n]* with unknown amplitude, polarity and length, *g[n]=w[n]+s[n]*. Depending on the knowledge of signal probability and distribution, different methods are available for detector selection and to determine a threshold level.

## 2. Methods

The Bayesian Risk estimation is a method that uses knowledge about previous detections for the decision process. As the probability of contamination is unknown, this is not applicable for our specific problem [1]. Therefore the Neyman-Pearson’s approach was used to determine the right detector and the associated threshold. The detector is decided by the likelihood ratio theorem *L(x)*, where *p* are the probability densities for the two states.

$$L(x) = \frac{p(x;H_1)}{p(x;H_0)} > \gamma$$

Using this approach we set the probability of a false alarm $P_{FA}$ to a fixed value and search for a detector which maximizes the probability of detection $P_D$. $P_{FA}$ is the probability that a detection ($H_1$) is made without a contaminant present ($H_0$) and is written as $P_{FA} = \Pr\{T(x) > \gamma; H_0\}$ and marked in Figure 1. The threshold is directly calculated from this value. $P_D$ is the probability of detecting passing samples, $P_D = \Pr\{T(x) > \gamma; H_1\}$ (Figure 2)[1].

If *s[n]* lasts only for one sample, STD is the optimal detector. If we include more information about the contaminant signal in the detection algorithm, we can expect better performance (higher $P_D$) from the detector. The energy detector should provide better results, as it sums the energy over *N* samples. We chose *N*=64 to encompass the majority of the energy (>99%) present in any given signal generated by a passing contaminant, which occurs in a length varying from 40 to 80 samples depending on conveyor speed and distance to the sensors.

### 2.1 Threshold calculation

The threshold $\gamma_1$ for the STD can be numerically calculated with $\gamma_1 = Q^{-1}(P_{FA}) \cdot \sqrt{\sigma^2} + \mu$ using the inverse function of *Q*. Here $\mu$ is the mean value and $\sigma^2$ the variance of *w[n]*.

$$Q(x) = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot \exp\left[-\frac{x^2}{2}\right] dx, \qquad P_{FA} = Q\left(\frac{\gamma - \mu}{\sqrt{\sigma^2}}\right),$$

*Q* is the right-tail-probability of the standard Gaussian distribution function starting from $\gamma$ as shown in Figure 1. For the energy detector the calculation becomes more complicated as the sum of squared Gaussian distributed values are Chi-squared distributed. But according to the central limit theorem we can approximate the distribution function to be Gaussian as we sum over 64 values. Thus the equations of the STD can be used to calculate the threshold.

### 2.2 Performance evaluation

To calculate $P_D$, and quantify the performance of a detector, we need to know the distribution of the contaminant signal, state $H_1$. This is difficult to do analytically as the signal will differ in amplitude, shape, frequency and polarity depending on the magnetic moment and the distance between the sample and the sensors. However, if a sample passes repeatedly through the system at the same position on the conveyor, we can extract the respective maxima of both the signal and the energy signal, caused by the contaminant and calculate their distribution function. We are now able to compare the detection performance of the detectors.

2.3 Sample preparation and data capturing

For performance evaluation it is important to use a sample close to the detection limit so that at least one detector experiences degradation of $P_D$ and the performance difference can be observed directly. A demagnetized 1x1x1 mm fraction of a hypodermic needle (stainless steel SS316) with a measured magnetic moment of 34 $n$Am$^2$ was used as a contaminant sample.

Data was captured from the running system with and without a contaminant cycling on a conveyor system using a DT9841E acquisition board from DataTranslation[2]. During a ten minute run the sample passed the sensors 64 times at a minimum distance of 8 cm and a velocity of approximately 0.35 m/s.

## 3. Results

Figure 2 shows models of $H_0$ and $H_1$ for the two detectors with parameters estimated from the captured data. Analysis of these models has shown that the ED performance is significantly better compared to the single threshold detector. For five thresholds, corresponding to $P_{FA}$ from $10^{-1}$ to $10^{-12}$ the detection probability for the energy detector was always above 99.9%, while for the STD, the $P_D$ degraded to 58.3% for a $P_{FA}$ of $10^{-12}$ (Table 1). The reason for the better performance of the ED is due to a low variance of the signal distribution function in the uncontaminated state compared to the mean signal energy, as short signal fluctuations are suppressed by summing the energy over multiple values.

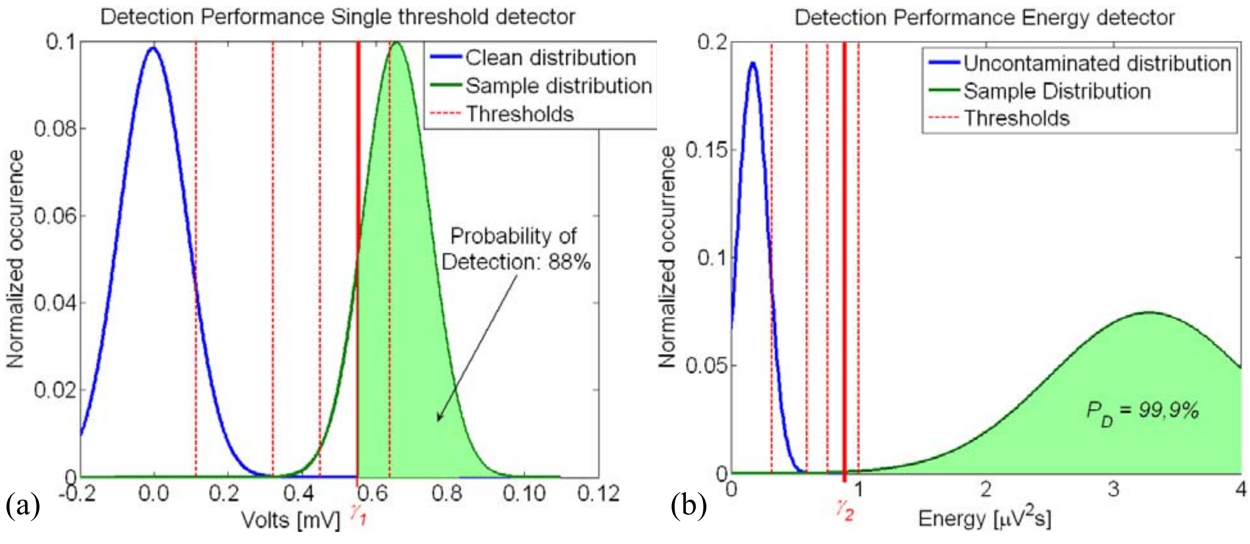


Figure 2. Probability density functions of (a) Single threshold detector (b) Energy detector. The vertical red lines mark the thresholds for $P_{FA}$ from $10^{-1}$ to $10^{-12}$ from left to right.

Table 1. Detector performance for five different thresholds.

| $P_{FA}$ | $P_D$ - STD | $P_D$ - ED |
|---|---|---|
| 1.00x $10^{-1}$ | 99.9 % | 100.0 % |
| 1.78x $10^{-4}$ | 99.9 % | 99.9 % |
| 3.16x $10^{-7}$ | 99.0 % | 99.9 % |
| 5.62x $10^{-10}$ | 88.0 % | 99.9 % |
| 1.00x $10^{-12}$ | 58.3 % | 99.9 % |

[2] http://www.datatranslation.com/products/dataacquisition/usb/dt9841e.asp

The distribution functions of both states for the STD show almost identical variance. This suggests that the amplitude of the contaminant signal is added on top of the normal signal fluctuation as expected, i.e. $g[n]=w[n]+s[n]$.

We applied the detector algorithms to the captured data to compare the results with the one from the models. The thresholds were chosen for a $P_{FA}$ of $10^{-12}$. Within the time interval shown in Figure 3 the sample passed the sensors 20 times. The STD was able to detect the sample only 10 times (green lines) whereas the ED managed to detect all 20 events.

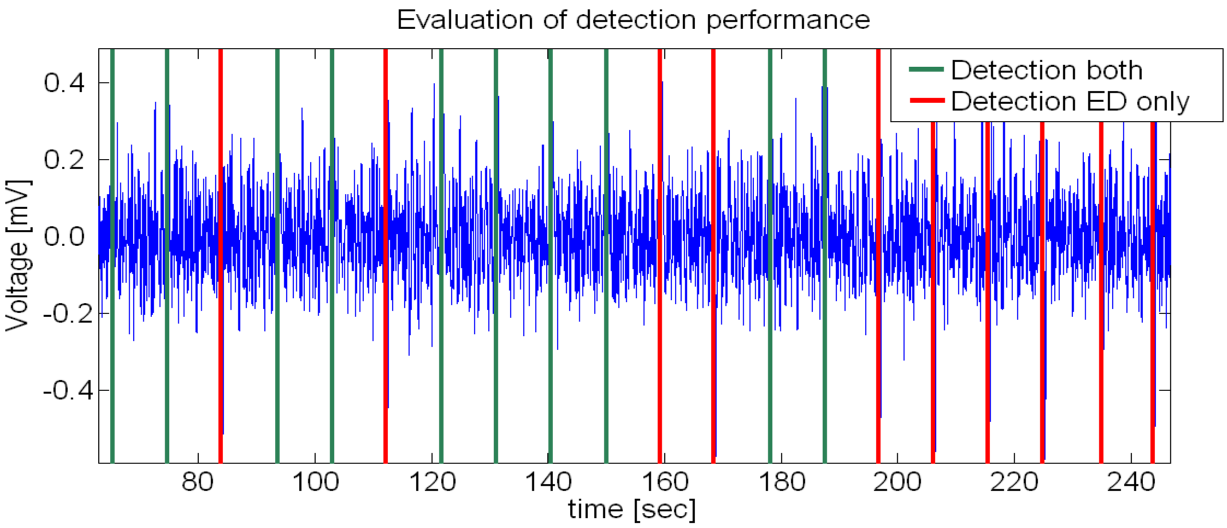


Figure 3. For $P_{FA} = 10^{-12}$ ED detected all 20 passes of the sample whereas the STD (green lines) detected the sample only 50% of the time.

We would set $P_{FA}$ as low as $10^{-8}$ to guarantee uninterrupted processing when no contaminant is present. For this we get 0.507 $m$V for the STD threshold and 0.836 $\mu V^2$s for ED. It is non-trivial to determine detection limits for samples of a given magnetic moment based on these thresholds, as the sensor response depends on distance and orientation of the sample, resulting in many possible energy levels for a given magnetic moment. However, based on the a required detection probability of 90%, reasonable estimates for the detection limits of the detectors for samples at a distance of 8 cm are 35 $n\mathrm{Am}^2$ for the STD and 15 $n\mathrm{Am}^2$ for the ED.

## Conclusions

During the analysis it was clear that the energy detector delivers a better detection performance, with an estimated detection limit of 15 $n\mathrm{Am}^2$, compared to the single threshold detector's estimate of 35 $n\mathrm{Am}^2$, at a distance of 8 cm. This is because the ED suppresses short signal fluctuations by summing the energy over multiple samples. The analytic results were confirmed by calculation of the signal distribution functions and detector tests on real data.

Theoretically an even better performance can be achieved when using a matched filter detector which maximizes the signal to noise ratio for the detection process, enhancing the detection performance over STD by a maximum of $\sqrt{N}$, where $N$ is the number of sample values.[1] The development of a matched filter detector is ongoing.


## Acknowledgments

The authors want to thank Marcel Bick, Keith Leslie, Chethasi Kaluarachchi, Gerard Degroot, Bob Thorn and Peter Sullivan for the help with the prototype system, measurement methods and their support for this project.


## References


[1] Steven M. Kay, *Fundamentals of Statistical Signal Processing, Volume 2: Detection Theory,* (Prentice Hall PTR, University of Rhode Island, 1998) p 61ff, 104.