

Density Matrix Renormalization Group study of the t - J_z Ladder

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The phase diagram of the two-leg t - J_z ladder is explored, using the density matrix renormalization group method. Results are obtained for energy gaps, electron density profiles and correlation functions for the half-filled and quarter-filled cases. A speculative phase diagram is presented, which is quite similar to the full t - J ladder, but scaled up by a factor of about two in coupling.

1. Introduction

The t - J_z model is a simplified model of strongly correlated itinerant electrons on a lattice, like the Hubbard and t - J models, which is of interest in connection with the cuprate high- T_c superconductors [1]. Here we study the model on a two-leg ladder (Figure 1). The Hamiltonian is

$$H = J \sum_i S_i^z S_{i+1}^z - t \sum_i (c_i^* c_{i+1} + \text{h.c.})$$

where $\{i,j\}$ are nearest-neighbour pairs of sites, t is the hopping parameter, and the J term is an Ising-type antiferromagnetic coupling between neighbouring electrons. Two electrons are forbidden to occupy the same site (on-site repulsion).

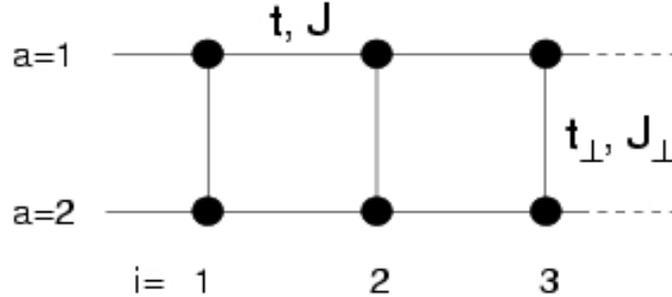


Fig 1. The t - J_z ladder

We treat the model using the density matrix renormalization group method, on ladders of up to 128 rungs. A selection of results is shown.

2. Near Half-Filling

At half-filling (one electron per site), the spins are Néel ordered with a finite spin gap, as in the Ising model. Figure 2 shows the binding energies for two-hole states for $S^z = 0, 1$, and samples of the electron density profiles. At $J = 0.5$ the holes are separate and unbound; at $J = 1$ they form a bound pair. Pairing in the $S^z = 1$ channel vanishes below $J=1.8$, and for the $S^z = 0$ channel below $J = 0.6$.

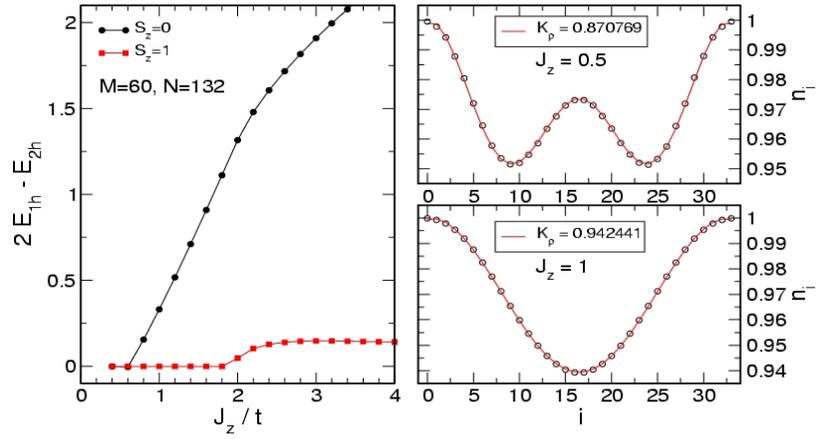


Fig. 2. Two-hole binding energies and electron density profiles at half-filling.

Figure 3 shows the four-hole binding energy, which is non-zero above $J = 4$, taken to indicate phase separation into hole-rich and hole-poor regions. The density profiles illustrate three regimes: 4 separate holes at $J = 0.5$, two pairs at $J = 1.0$ and a 4-hole cluster at $J = 6.0$.

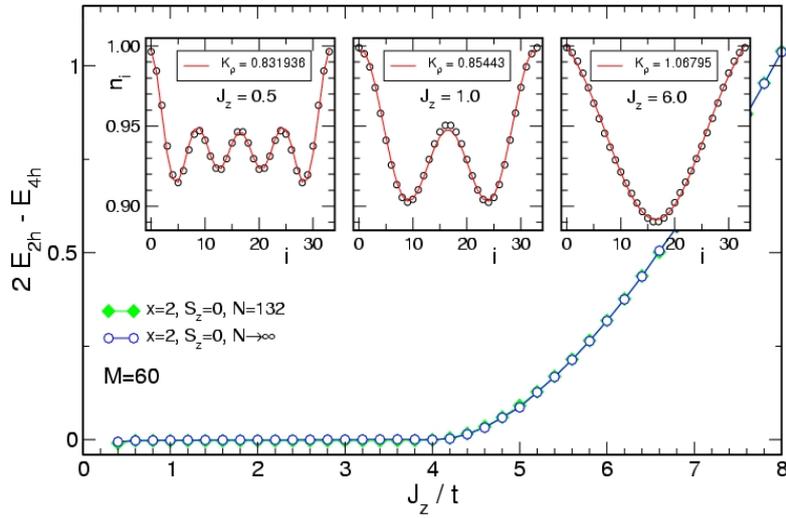


Fig.3. Four-hole binding energy and density profiles at half-filling.

3. Near Quarter-Filling

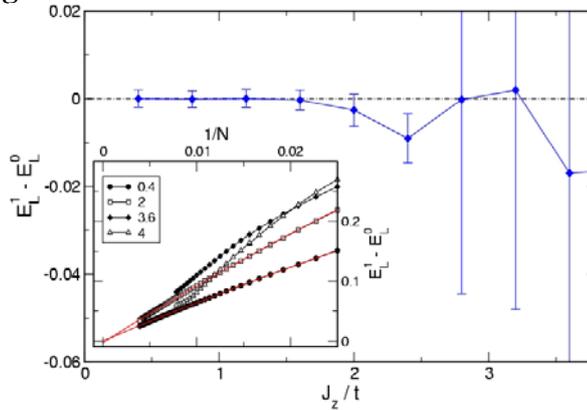


Fig. 4. The spin gap at quarter-filling.

At quarter-filling, results are less reliable. Figure 4 shows the spin gap, compatible with zero everywhere. Figure 5 shows the estimated two-hole gap, apparently zero beyond $J = 2.4$. The correlation functions are solid-like for smaller J , compatible with a *charge density wave* state, but liquid-like for larger J , possibly indicating the onset of phase separation.

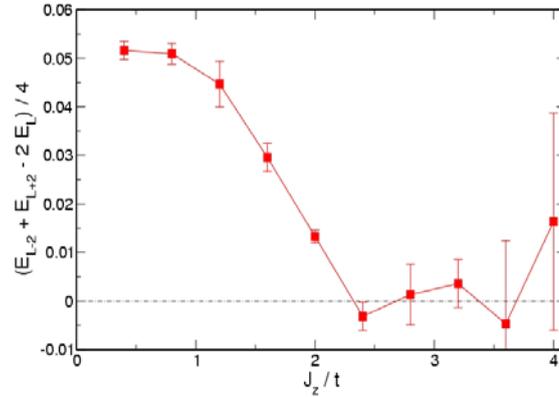


Fig. 5. The two-hole gap at quarter-filling.

Conclusions

The phase diagram for the model appears very similar to that of the full t - J ladder [2-5]. Near half-filling, there is a Luther-Emery phase with gapless charge excitations and gapped spin excitations, which is expected to show superconducting correlations. The effective velocity parameter v_ρ is found to vanish at half-filling. Near quarter-filling, there is a Tomonaga-Luttinger phase, with one gapless charge and one gapless spin excitation. At commensurate filling factors, a charge density wave state appears at low J . Question: is there a Nagaoka phase at small J near half-filling, and if so, what is the behaviour in that phase?

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