

What causes dissipation in a ballistic quantum point contact?

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Background

The understanding of mesoscopic transport and noise was established in the last decade through the insights of Landauer, Büttiker, Imry, and many other contributors [1, 2]. That succinct, and very successful, phenomenology identifies the basis of mesoscopic current flow as a mismatch of carrier density between the metallic reservoirs (terminals) across which the low-dimensional conductor (quantum point contact) of actual interest is connected. We are dealing here with small – even quasi-molecular – structures. Consequently they experience a high degree of *openness* to their macroscopic environment.

A striking signature of transport in quantum point contacts is the discretization of conductance into "Landauer steps", in units of $2e^2/h \approx 0.078\text{mS}$. They are explained via collisionless quantum transmission of individual electrons through a one-dimensional, lossless barrier. However, simple quantum-coherent scattering cannot tackle the central issue of conduction: *What causes dissipation in a ballistic quantum point contact?*

The question is far more than academic. In the near future, reliable and effective nano-electronic design will demand not merely fashionable models, but ones that are credible. It is beyond the gift of coherence-based phenomenologies to cover the physics that is needed.

Below we outline the answer to our question. It is given uniquely by many-body quantum kinetics [3]. Our microscopic application of many-body methods leads not only to conductance quantization by fully accounting for inelastic energy loss [4], but we also resolve a long-standing experimental enigma [5] in the noise spectrum of a quantum point contact (QPC) [6]. The same developments foreshadow a systematic pathway to truly predictive design of novel structures.

The Physical Issue

The core issue in the physics of conduction is plain to state. Any finite conductance G must dissipate electrical energy at the rate $P = IV = GV^2$, where $I = GV$ is the current and V the potential difference across the terminals of the driven conductor. It follows that there must be an explicit physical mechanism (e.g. emission of optical phonons) by which the energy gained by carriers, when transported from source to drain, is channelled to the surroundings. Alongside any elastic and coherent scattering processes, inelastic processes must always be in place. Harnessed together, they fix G ; yet it is only the energy-dissipating mechanisms that secure the *thermodynamic stability* of steady-state conduction.

There is a complete microscopic understanding of the ubiquitous power-loss formula $P = GV^2$ [7, 8]. It resides in the fluctuation-dissipation theorem, valid for *all* resistive devices

at all scales, without exception. The theorem expresses the requirement for thermodynamic stability. With it comes the conclusion that [3,4]

- inelasticity is necessary and sufficient to stabilize current flow at finite conductance;
- ballistic quantum point contacts have finite $G \propto 2e^2/h$; therefore
- the physics of energy loss is indispensable to a proper theory of ballistic transport.

The physics of explicit inelastic scattering is beyond the scope of transport models that rely only on coherent quantum scattering to explain the origin of G in quantum point contacts. Coherence implies elasticity, and elastic scattering is always loss-free: it conserves the energy of the scattered particle. This reveals the deficiency of purely elastic models of transmission. We now review a well-defined microscopic remedy for this deficiency.

The Physical Solution

To allow for the energy dissipation vital to any microscopic description of ballistic transport, we recall that open-boundary conditions imply the intimate coupling of the QPC channel to its interfaces with the reservoirs. The interface regions must be treated as an integral part of the device model. They are the very sites for strong scattering effects: *dissipative* many-body events as the current enters and leaves the ballistic channel, and *elastic* one-body events as the carriers interact with background impurities, the potential barriers that confine and funnel the current, and so on.

The key idea in our treatment is to subsume the interfaces within the total kinetic description of the ballistic channel. At the same time, strict charge conservation in an *open* device requires the direct supply and removal of current by an external generator [8]. Thus the current cannot depend on the physics of the local reservoirs. This canonical requirement sets the quantum kinetic approach entirely apart from Landauer-like treatments [1], which rest upon the phenomenological notion that the current depends on density differences between reservoirs.

(a) Ballistic Conductance

It is straightforward to write the algebra for the conductance in our model system. A uniform, one-dimensional ballistic QPC, of operational length L , will be associated with two mean free paths determined by v_F , the Fermi velocity of the electrons, and a pair of characteristic scattering times τ_{el} , τ_{in} . Thus

$$\lambda_{el} = v_F \tau_{el}; \quad \lambda_{in} = v_F \tau_{in}. \quad (1)$$

Respectively, these are the scattering lengths set by the elastic and inelastic processes active at both interfaces. The device (the QPC *and* its interfaces) has a conductive core that is collisionless. It follows that

$$\lambda_{el} = L = \lambda_{in}. \quad (2)$$

Finally, the channel's conductance is given by the familiar formula

$$G = \frac{ne^2 \tau_{tot}}{m^* L} = \frac{2k_F}{\pi} \frac{e^2}{m^* L} \left(\frac{\tau_{in} \tau_{el}}{\tau_{el} + \tau_{in}} \right); \quad (3)$$

the effective mass of the carriers is m^* . In the first factor of the rightmost expression for G we rewrite the density n in terms of the Fermi momentum k_F ; in the final factor, we use Matthiessen's rule $\tau_{tot}^{-1} = \tau_{el}^{-1} + \tau_{in}^{-1}$ for the total scattering rate in the system.

Using Equations (1)–(3), the conductance reduces to

$$G = 2 \frac{e^2}{\pi \hbar} \frac{\hbar k_F}{m^* L} \left(\frac{(L/v_F)^2}{2L/v_F} \right) = \frac{2e^2}{h} \equiv G_0. \quad (4)$$

This is precisely the Landauer conductance of a single, one-dimensional, ideal channel.

None of the adventitious assumptions, otherwise invoked to explain conductance quantization [1, 2], has been used. Indeed, the result emerges from completely standard quantum kinetics. Most important is the clear and central role of inelastic energy loss, one of the underpinnings of quantum transport. Charge conservation, the other underpinning, is guaranteed by our use of microscopically consistent open-boundary conditions at the interfaces. These physical requirements are not transparent in more phenomenological derivations of Eq. (4).

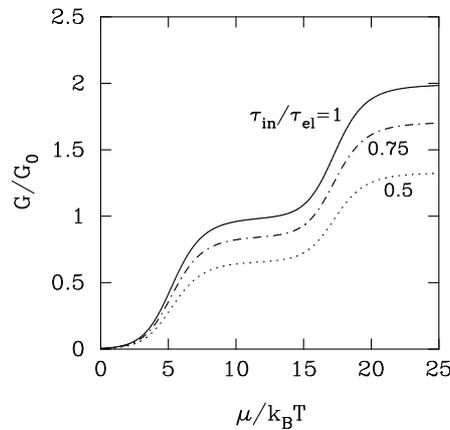


Figure 1: Conductance quantization in a two-band ballistic point contact, as a function of chemical potential μ , calculated with our kinetic theory (see Ref. [3]). Full curve: ideal ballistic channels. Broken curves: non-ideal behaviour increases with the onset of inelastic phonon emission inside the contact.

In Figure 1 we plot the results of our model for a QPC [3] made up of two one-dimensional conduction bands at energies $5k_B T$ and $17k_B T$, in thermal units at temperature T . We use the natural extension of Eq. (4) to cases where one or more channels may be open to conduction, depending on T and the size of the chemical potential μ . As the role of inelastic scattering is enhanced ($\tau_{in} < \tau_{el}$) the conductance deviates from the ideally ballistic Landauer limit.

(b) *Non-equilibrium Noise*

The noise response of a quantum point contact is a fascinating aspect of mesoscopic transport, and a more demanding one both experimentally and theoretically. In 1995, a landmark measurement of nonequilibrium noise was performed by the Weizmann group [5], which yielded a very puzzling result. Whereas conventional models [2] predicted a strictly monotonic shot-noise signal for a QPC driven at constant current levels, the data showed a series of very strong peaks at threshold (where the carrier density in the QPC rapidly grows and becomes metallic). This is in marked contradiction to theoretical expectations.

As remarkable as they were, the Weizmann results remained absolutely unexplained for a

decade. We have now accounted for them within our strictly conserving kinetic description [6].

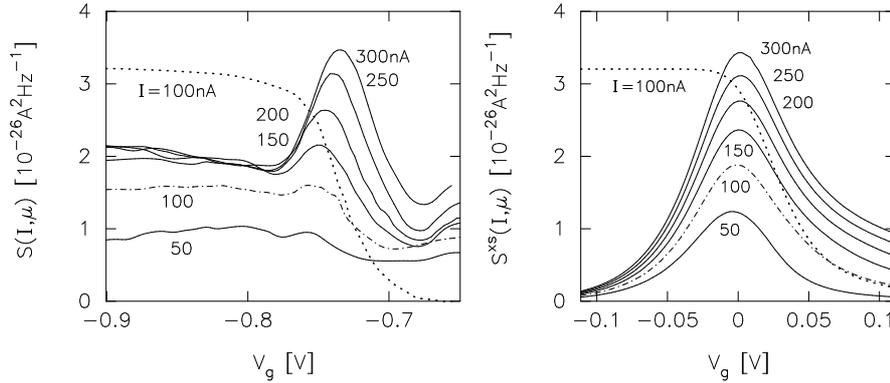


Figure 2: Nonequilibrium current noise of a QPC at constant source-drain current, as a function of gate bias. Left: data from Reznikov *et al.*, Ref. [5]. Right: calculation from Green *et al.*, Ref. [6]. In each case the dotted line traces the standard shot-noise prediction at 100nA using, as respective inputs, measured and calculated data for G . The standard prediction is well wide of the mark.

In Fig. 2 we display the experimental data side by side with our computation of excess QPC noise under the same conditions [6]. Note the close accord between measurement and calculation. This is in contrast to the outcome of standard phenomenologies [2].

Summary

The kinetic approach to transport provides a fully microscopic account of conductance and noise in quantum point contacts. We accurately reproduce the current response of mesoscopic conductors, in particular conductance quantization. *Open-system charge conservation* and the reality of *dissipative scattering* are the keys to this new and fruitful physical picture.

Our unified theory yields a detailed understanding of the fundamental nonequilibrium fluctuations in a QPC. We have successfully tested this understanding by fully explaining the long-standing puzzle posed by the noise measurements of Reznikov *et al.* [5]. Noise and fluctuations carry much more information on the internal dynamics of mesoscopic systems – knowledge that is not accessible through the I - V characteristics alone.

The capacity to build up a mesoscopic theory which is truly systematic, not thrown together *ad hoc*, encompasses huge potential to improve the practical design of new generations of devices. These can – and clearly should – be built on a solid, non-speculative, knowledge base.

References

- [1] Y. Imry, *Introduction to Mesoscopic Physics*, 2nd ed (OUP, Oxford, 2002).
- [2] Y. M. Blanter and M. Büttiker, *Phys. Rep.* **336**, 1 (2000).
- [3] M. P. Das and F. Green, *J. Phys.: Condens. Matter* **15**, L687 (2003).
- [4] F. Green and M. P. Das, *J. Phys.: Condens. Matter* **12**, 5233 (2000); *ibid*, 5251.
- [5] M. Reznikov *et al.*, *Phys. Rev. Lett.* **75**, 3340 (1995).
- [6] F. Green *et al.* *Phys. Rev. Lett.* **92**, 156804 (2004).
- [7] R. Kubo, M. Toda, and M. Hashitsume, *Statistical Physics II: Nonequilibrium Statistical Mechanics*, 2nd ed (Springer, Berlin, 1991).
- [8] F. Sols, *Phys. Rev. Lett.* **67**, 2874 (1991); W. Magnus and W. Schoenmaker, *Quantum Transport in Sub-micron Devices: A Theoretical Introduction* (Springer, Berlin, 2002).