Weak Measurements on Entangled Solid-State Qubits

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We show that the singlet and triplet states of two electrons on a double quantum dot (DQD) can be entangled with the same states of a second DQD. Since weak measurements (WMs) can be performed on the relevant states of DQDs, we can investigate correlations between distant WMs. Unlike strong measurements, WMs do not exhibit nonclassical statistical correlations.

1. Introduction

A normal (or projective) measurement of an observable $\hat{O}$ in quantum mechanics involves a strong coupling between the quantum system (prepared in the state $|\psi_i\rangle$) and a measuring system so that the outcome recorded in the measurement identifies one of the eigenvalues of $\hat{O}$ and the quantum system is left in the corresponding eigenstate at the end of the measurement. In a *weak measurement* (WM), there is only very weak coupling between the quantum system and the measuring instrument with the result that the outcome recorded in the measurement gives information about $\langle\psi_o|\hat{O}|\psi_o\rangle$ and the state of the quantum system is known only imprecisely at the end of the measurement. The term WM is closely related to, but not to be confused with, the *weak value* (WV) [1] which applies to a *sub-ensemble* conditionalised on the outcome $|\psi_f\rangle$ of a subsequent projective measurement of a different observable. The average of the WM results over the post-selected ensemble gives the *weak value* (WV) of $\hat{O}$, $\langle\hat{O}\rangle_W = \langle\psi_f|\hat{O}|\psi_i\rangle / \langle\psi_f|\psi_i\rangle$ (the real or imaginary parts can be measured experimentally). Weak values have unusual properties, for example $\langle\hat{O}\rangle_W$ may lie outside the eigenvalue spectrum of $\hat{O}$. There is increasing interest in WMs (and WVs) of solid-state qubits because of their quantum information applications [2]. Recently it has been shown how to determine the WV of the projector onto the singlet ground state of a double quantum dot (DQD) [3]. Strong measurements on entangled electron spins have been used to demonstrate the remarkable properties of nonlocality and nonseparability in quantum mechanics. The aim of the present work is to investigate for the first time whether WMs on entangled electron spins also exhibit those phenomena.

2. Weak measurements on quantum dot

The experimental arrangement proposed by Romito et al [3] is shown in Fig. 1. Two electrons occupy the DQD and the occupancies of the left dot (n_L) and the right dot (n_R) are controlled by the gate voltages V_L and V_R respectively. Only two charge states (n_L, n_R) = (1,1) and (0,2) are involved with, respectively, energies $E(1,1) = E_0(V_L + V_R)$ and $E(0,2) = 2E_0V_R$ where E_0 is a constant. If the energy zero is $E_0(V_L + 3V_R)/2$ then $E(1,1) = +\varepsilon E_0$ and

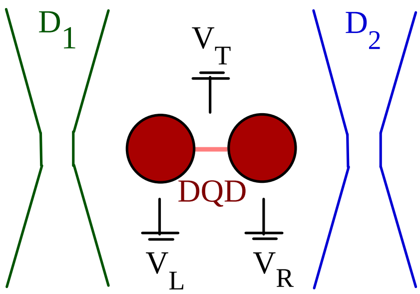

Fig. 1. The double quantum dot (DQD) with spin states controlled by the gate voltages V_L, V_R and V_T. Either of the detectors D_1 or D_2 can be used to distinguish between the charge states (1,1) and (0,2) of the DQD.

$E(0,2) = -\varepsilon E_0$ where $\varepsilon = (V_R - V_L)$. The spin states involved are: the singlet states $|\mathrm{S}(1,1)\rangle \equiv |\mathrm{S}\rangle = (|\uparrow\rangle_R|\downarrow\rangle_L - |\downarrow\rangle_R|\uparrow\rangle_L)/\sqrt{2}$, $|\mathrm{S}(0,2)\rangle \equiv |\bar{\mathrm{S}}\rangle = (|\uparrow\rangle_R|\downarrow\rangle_R - |\downarrow\rangle_R|\uparrow\rangle_R)/\sqrt{2}$ and triplet states $|\mathrm{T}_-(1,1)\rangle \equiv |\mathrm{T}\rangle = (|\uparrow\rangle_R|\downarrow\rangle_L + |\downarrow\rangle_R|\uparrow\rangle_L)/\sqrt{2}, |\mathrm{T}_+(1,1)\rangle \equiv |\mathrm{T}_+\rangle = |\uparrow\rangle_R|\uparrow\rangle_L, |\mathrm{T}_-(1,1)\rangle \equiv |\mathrm{T}_-\rangle = |\downarrow\rangle_R|\downarrow\rangle_L$ (the $|\mathrm{T}(0,2)\rangle$ states are too high in energy to be occupied). An external magnetic field $B_{\mathrm{ext}}\hat{\mathbf{z}}$ splits off the $|\mathrm{T}_+\rangle$ and $|\mathrm{T}_-\rangle$ states which can be ignored from now on for simplicity without materially affecting the results. The gate voltage V_T controls tunnelling between the two singlet states with tunnelling amplitude λ. The Hamiltonian for the DQD is $\hat{H} = \varepsilon E_0 (|T\rangle\langle T| + |S\rangle\langle S| - |\bar{S}\rangle\langle\bar{S}|) + \lambda E_0 (|S\rangle\langle T| + |T\rangle\langle S|)$ with eigenstates and eigenenergies

$$|T\rangle, \qquad E_T = \varepsilon E_0 \tag{1a}$$

$$|S_g\rangle = \cos\theta_\varepsilon |S\rangle - \sin\theta_\varepsilon |\bar{S}\rangle, \quad E_g = -E_0\sqrt{\varepsilon^2 + \lambda^2} \tag{1b}$$

$$|S_e\rangle = \sin\theta_\varepsilon |S\rangle + \cos\theta_\varepsilon |\bar{S}\rangle, \quad E_e = +E_0\sqrt{\varepsilon^2 + \lambda^2} \tag{1c}$$

where $\tan\theta_\varepsilon = \left(\varepsilon + \sqrt{\varepsilon^2 + \lambda^2}\right)/\varepsilon$. The energy level scheme is shown in Fig. 2. Note that for $\varepsilon \ll \varepsilon_B < 0$, the charge state of the lower energy singlet state $|S_g\rangle$ is (1,1) ($|S\rangle$) and for $\varepsilon \gg \varepsilon_A > 0$, the charge state of $|S_g\rangle$ is (0,2) ($|\bar{S}\rangle$).

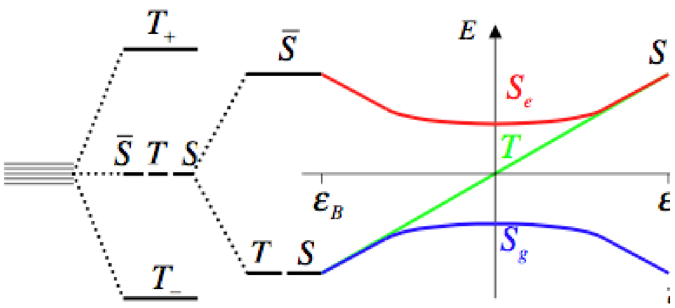

Fig.2. Energy level scheme for each DQD. The degeneracy is removed firstly by an external magnetic field and then by the gate voltages V_T and $V_R - V_L \propto \varepsilon$. [The states T_+, T_- are omitted from the right hand part of the figure for clarity.]

The WM is performed over the period τ by the quantum point contact (QPC) D_1 responding to the charge state of the left dot of the DQD. If the left dot is occupied [charge state (1,1)], the state $|\mathrm{in}\rangle$ of a carrier approaching D_1 evolves to $|\varphi_o\rangle = t_o|t\rangle + r_o|r\rangle$ where $|t\rangle$ and $|r\rangle$ are the transmitted and reflected states of the carrier while if the left dot is unoccupied [charge state (0,2)], the carrier evolves to $|\varphi_u\rangle = t_u|t\rangle + r_u|r\rangle$. The WM is represented by the operator $|m\rangle\langle m|\hat{U}(\tau)$, $m = t$ or r where

$$\begin{aligned}\hat{U}(\tau) &= |\bar{S}\rangle\langle\bar{S}| \otimes |\varphi_u\rangle\langle\mathrm{in}| + (|S\rangle\langle S| + |T\rangle\langle T|) \otimes |\varphi_o\rangle\langle\mathrm{in}| \\ &= |\bar{S}\rangle\langle\bar{S}| \otimes |\varphi_u\rangle\langle\mathrm{in}| + (\hat{I}_{DQD} - |\bar{S}\rangle\langle\bar{S}|) \otimes |\varphi_o\rangle\langle\mathrm{in}|\end{aligned} \tag{2}$$

where $\hat{I}_{DQD}$ is the identity operator in the Hilbert space of the DQD. The spin system is prepared in a state not involving $|S_e\rangle$ and, to a good approximation, transitions to $|S_e\rangle$ are not allowed under the conditions of the experiment. Thus, from Eq. (1b), $|\bar{S}\rangle\langle\bar{S}| \approx \sin^2\theta_\varepsilon\, |S_g\rangle\langle S_g|$, where $\sin^2\theta_\varepsilon = \left(\varepsilon + \sqrt{\varepsilon^2 + \lambda^2}\right)/2\sqrt{\varepsilon^2 + \lambda^2}$, so the measurement by D_1 can be made arbitrarily weak $\left(\hat{U}(\tau) \to \hat{I}_{DQD}\right)$ by the choice of ε and λ.

We will be concerned with entangled states involving the $|S_g\rangle$ and $|T\rangle$ states (see Eq. (4) below) and in order to examine the statistical correlations between them, it is necessary to be able to manipulate them like the spin states in a normal Bell state. This can be done because the hyperfine interaction between the electron spin and the nuclear spins on each quantum dot drive rotations between $|S_g\rangle$ and $|T\rangle$. The hyperfine interaction is described by the Hamiltonian $\hat{H}_N^{\mathrm{J}} = \hbar\omega_{\mathrm{hf}}\left(|T\rangle^{\mathrm{J}}\langle S_g|^{\mathrm{J}} + |S_g\rangle^{\mathrm{J}}\langle T|^{\mathrm{J}}\right)$ where $\hbar\omega_{\mathrm{hf}}^{\mathrm{J}} = g\mu_B\left(\mathbf{B}_{NR}^{\mathrm{J}} - \mathbf{B}_{NL}^{\mathrm{J}}\right)\cdot\hat{\mathbf{z}}$ and $\mathbf{B}_{NR}^{\mathrm{J}}, \mathbf{B}_{NL}^{\mathrm{J}}$ are the magnetic fields due to the nuclear spins in the respective dots, g is the g-value of the electron spins and μ_{B} is the Bohr magneton. During time t, the interaction results in the following time evolution of the states

$$
\begin{aligned}
e^{-i\hat{H}_N^{\mathrm{J}} t}|S_g\rangle^{\mathrm{J}} &= \cos\alpha^{\mathrm{J}}|S_g\rangle^{\mathrm{J}} - i\sin\alpha^{\mathrm{J}}|T\rangle^{\mathrm{J}} \\
e^{-i\hat{H}_N^{\mathrm{J}} t}|T\rangle^{\mathrm{J}} &= -i\sin\alpha^{\mathrm{J}}|S_g\rangle^{\mathrm{J}} + \cos\alpha^{\mathrm{J}}|T\rangle^{\mathrm{J}}
\end{aligned} \tag{3}
$$

where $\alpha^{\mathrm{J}} = \omega_{\mathrm{hf}}^{\mathrm{J}}\, t$. We next consider the initial and final states so that we can calculate the weak value for an entangled state involving two DQDs (labelled J = A, B) of the above type.

3. Initial and final states

Initially, the tunnelling in each DQD is turned off and ε = ε_{B} so that $|S_g\rangle = |S\rangle$ and $|\uparrow\rangle_R|\downarrow\rangle_L$, $|\downarrow\rangle_R|\uparrow\rangle_L$ are the degenerate lowest energy states for the dot. At $t=0$, the spins on each of the left dots are entangled, for example by the method described in reference [4], and the right dots are entangled in a different Bell state. As a result, the singlet and triplet states on each DQD become entangled:

$$
|\psi;0\rangle = \frac{1}{2}\left(|\uparrow\rangle_L^{\mathrm{A}}|\downarrow\rangle_L^{\mathrm{B}} - |\downarrow\rangle_L^{\mathrm{A}}|\uparrow\rangle_L^{\mathrm{B}}\right)\left(|\uparrow\rangle_R^{\mathrm{A}}|\downarrow\rangle_R^{\mathrm{B}} + |\downarrow\rangle_R^{\mathrm{A}}|\uparrow\rangle_R^{\mathrm{B}}\right) = \frac{1}{2}\left(|S_g\rangle^{\mathrm{A}}|T\rangle^{\mathrm{B}} - |T\rangle^{\mathrm{A}}|S_g\rangle^{\mathrm{B}} + |T_+\rangle^{\mathrm{A}}|T_-\rangle^{\mathrm{B}} - |T_-\rangle^{\mathrm{A}}|T_+\rangle^{\mathrm{B}}\right)
$$

As before, we will drop the last two terms for clarity. As a result of the nuclear hyperfine evolution, the initial (pre-selection) state of the DQD at t is

$$
|\psi_i\rangle = \frac{1}{\sqrt{2}}\left(\cos\left(\alpha^{\mathrm{A}} - \alpha^{\mathrm{B}}\right)\left(|S_g\rangle^{\mathrm{A}}|T\rangle^{\mathrm{B}} - |T\rangle^{\mathrm{A}}|S_g\rangle^{\mathrm{B}}\right) + i\sin\left(\alpha^{\mathrm{A}} - \alpha^{\mathrm{B}}\right)\left(|S_g\rangle^{\mathrm{A}}|S_g\rangle^{\mathrm{B}} - |T\rangle^{\mathrm{A}}|T\rangle^{\mathrm{B}}\right)\right) \tag{4}
$$

A WV requires selection of the final state which can done by imposing a fast adiabatic variation of ε (< 0) used during the WM to ε = ε_{A} for each DQD. The $|S_g\rangle$ state of the quantum system on each DQD is then $|\bar{\mathrm{S}}\rangle = |\mathrm{S}(0,2)\rangle$ which can be probed by a projective measurement by QPC $\mathrm{D}_2^{\mathrm{J}}$. To form the pre- and post-selected ensemble (PPSE), the results of the WM (by $\mathrm{D}_1^{\mathrm{J}}$) are only retained if $\mathrm{D}_2^{\mathrm{J}}$ records $|\bar{\mathrm{S}}\rangle^{\mathrm{J}} \equiv |S_g\rangle^{\mathrm{J}}$ at $\varepsilon = \varepsilon_A$. If the fast adiabatic steps are made at a time t_f after the WM, then due to the hyperfine interaction, the final (post-selection) state at the end of the WM is

$$
\begin{aligned}
|\psi_f\rangle &= \cos\beta^1\cos\beta^2|S_g\rangle^1|S_g\rangle^2 \\
&\quad + i\left(\cos\beta^1\sin\beta^2|S_g\rangle^1|T\rangle^2 + \sin\beta^1\cos\beta^2|T\rangle^1|S_g\rangle^2\right) - \sin\beta^1\sin\beta^2|T\rangle^1|T\rangle^2
\end{aligned} \tag{5}
$$

where $\beta^{\mathrm{J}} = \omega_{\mathrm{hf}}^{\mathrm{J}}\, t_f$. We have now established (i) a source of entangled states involving $|S_g\rangle$ and $|T\rangle$, (ii) a means of rotating states in the Hilbert space spanned by those states, (iii) performing a weak measurement of $|S_g\rangle\langle S_g|$ and (iv) pre-and post-selecting states.

4. Results

Using Bayes' rule, the probability $P\left(t\,\&\,t\middle|\psi_i,\psi_f\right)$ that D_1^A and D_1^B both register a transmitted outcome for the PPSE is

$$P\left(t\,\&\,t\middle|\psi_i,\psi_f\right)=\frac{P\left(t\,\&\,t\middle|\psi_i\right)P\left(\psi_f\middle|\psi_i,t\,\&\,t\right)}{P\left(\psi_f\middle|\psi_i\right)}=\frac{\left|\langle tt|U(\tau)|\psi_i\rangle\right|^2\left|\langle\psi_f|\psi_i^{tt}\rangle\right|^2}{\sum_{m,n=t,r}\left|\langle mn|U(\tau)|\psi_i\rangle\right|^2\left|\langle\psi_f|\psi_i^{mn}\rangle\right|^2}$$

where $\left|\psi_i^{mn}\right\rangle=\langle mn|U(\tau)|\psi_i\rangle/\left|\langle mn|U(\tau)|\psi_i\rangle\right|$. For WMs $\sin^2\theta_\varepsilon \ll 1$ (we assume the same ε for both DQDs) and in that limit

$$P\left(t^A\,\&\,t^B\middle|\psi_i,\psi_f\right)\approx|t_o|^2\left(|t_o|^2+2\sin^2\theta_\varepsilon\,\mathrm{Re}\left[t_o^*\delta t\left\langle\hat{S}^A\right\rangle_W+t_o^*\delta t\left\langle\hat{S}^B\right\rangle_W\right]\right)$$

where the weak value $\left\langle\hat{S}^J\right\rangle_W=\left\langle\psi_i\right|\left(|S_g\rangle^J\langle S_g|^J\right)\left|\psi_f\right\rangle/\langle\psi_i|\psi_f\rangle$ and $\delta t=t_0-t_u$. In the present case, $\left\langle\hat{S}^A\right\rangle_W=\cos\beta^A\sin\left(\alpha^A-\alpha^B-\beta^B\right)/\cos\left(\alpha^A-\alpha^B+\beta^A-\beta^B\right)$ and $\left\langle\hat{S}^B\right\rangle_W=\cos\beta^B\sin\left(\alpha^A-\alpha^B+\beta^A\right)/\cos\left(\alpha^A-\alpha^B+\beta^A-\beta^B\right)$. The probability that D_1^J alone registers a transmitted outcome is

$$P\left(t^J\middle|\psi_i,\psi_f\right)=P\left(t^Jr^{J'}\middle|\psi_i,\psi_f\right)+P\left(t^Jt^{J'}\middle|\psi_i,\psi_f\right)\approx|t_o|^2+2\sin^2\theta_\varepsilon\,\mathrm{Re}\left[t_o^*\delta t\left\langle\hat{S}^J\right\rangle_W\right]$$

which is the same as the corresponding expression found in Ref. [3] for a single DQD.

The important result is that, to the level of approximation used in dealing with WVs (ignore terms in $\sin^n\theta$ for $n>2$), $P\left(t^1\,\&\,t^2\middle|\psi_i,\psi_f\right)=P\left(t^1\middle|\psi_i,\psi_f\right)P\left(t^2\middle|\psi_i,\psi_f\right)$ which means the transmission probabilities on the two DQDs are statistically uncorrelated, i.e. they satisfy *classical* correlation conditions. This is in stark contrast to strong measurements on normal (pre-selected only) ensembles which exhibit *nonclassical* correlations. It is known the correlations for strong measurements for PPSEs are also nonclassical [5].

5. Discussion and conclusion

It is the *nonclassical* statistical correlations of strong measurements on entangled systems that leads to the puzzling properties of nonlocality and nonseparability characteristic of quantum systems. The present result means that those puzzling properties do not apply to WVs for entangled states. It has recently been shown, using solid-state qubits, that the Leggett-Garg inequalities (which apply to a single and therefore non-entangled system) can be nonclassical for WVs [6]. Therefore, correlations among WVs are not classical in general and it may be that it is the particular property of nonlocality which does not apply to WVs.

It has been shown here that the singlet and triplet states of each of two DQDs can be entangled and the weak values of the singlet states, measured by the method based on Ref. [3], do not exhibit nonclassical statistical correlations. One consequence of the latter conclusion means that weak values could not be used directly in quantum computation and quantum information applications which rely on nonclassical statistical correlations.

References

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